Course: STA 124 – Introduction to Probability Distribution
(2 Credits – Compulsory)
Course Duration: Two hours per week for 15 weeks (30 hours)
As taught in 2011/2012 session

Lecturer: **ADEJUMO,** Adebowale Olusola
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Office Location: –F4, Department of Statistics, Statistics Building, Faculty of Science.
Consultation Hours: 2:30 – 4:00pm Tuesdays & Thursdays.

Course Content:

Random variable, Bermoulli trials, Binomial, Geometric, Poisson, Uniform and Normal distributions. Concepts of linear regression, correlation and association of attributes.
30h(T); C

Course Description:

As introduction, we shall consider what random variable is in details, as well as the types of random variables. The course introduce the students to the concepts of probability: experiment, outcome, trial, chance, event and sample space, Expectations, variance, distribution functions; Bivariate distribution: marginal distribution, conditional distribution; Probability distributions: Bernoulli, Binomial, Poisson, Geometric, Uniform and Normal distributions. Regression: concept of Simple linear equation, Analysis of variance for regression; Correlation: Pearson correlation, Rank correlation. Association of attributes.

Course Justification:

The course is designed to introduce students in the Mathematical Sciences, Physical Sciences and Engineering to the application of statistics in their discipline. The course is to intimate the students with the usefulness of statistics in their various fields of studies and enlighten the student on the importance of statistics in carrying out researches in their various areas of study. It is also to develop in students, the ability to apply their knowledge and skills to the solution of theoretical and practical problems in Statistics.

Course Objectives:

The general objective of this course is for the students to know how to use statistical tools in their field of studies. To intimate the student with all necessary statistical measures and techniques of deriving reasonable conclusion on any experiment carried out in their field of studies. In addition, it is to instil in students, a sense of enthusiasm for Statistics, an appreciation of its application in different areas and to involve them in an intellectually stimulating and satisfying experience of learning and studying.

At the end of the course, the students will be able to know:

- different random variables, and how to generate them,
- the concepts of probability,
- the concepts of Bivariate distribution, marginal and conditional distributions,
- different probability distributions,
- how to fit a simple linear model,
- how to fit a simple ANOVA table for simple linear model,
- the concept of correlation and
- the concepts of association of attribute.

Course Requirements:

- This is a compulsory course for students in Departments of Mathematics, Computer Science, Physics, Engineering, Mathematics Science Education and Statistics. Students are expected to participate in all the course activities and have minimum of 75% attendance to be able to write the final examination.
- They will also be expected to treat the study questions and assignments.
- Students are also expected to have e-mail accounts.

Methods of grading:

No	Item	Score %
1.	Class assignments/ test (likely on CBT)	30
2.	Comprehensive final examination (likely on CBT)	70
	Total	**100**

Course Delivery Strategies:

The lecture will be delivered through face-to-face method, theoretical material (lecture note) provided during lecture with working examples to demonstrate the theory lessons.
Students will be encouraged and required to read around the topics and follow current issues in the media. Web-interactions will be employed by requesting each student to have yahoo e-mail address to enable them participate in the yahoo discussion group that had been created for the course (ao123adejumo@yahoo.co.uk). Additional materials and links will be provided on the board. The delivery strategies will also be supported by tutorial sessions and review of study questions.

Reading List:

[1]Oyejola, B. A. and Adebayo, S. B. (2004). Basic Statistics for Biology and Agriculture Students. OLAD Ilorin.

[1]Nduka E. C and Ogbonnaya, C. I. (1996). Fundamentals of Experimental Statistics. FASMEN Okigwe.

LECTURE CONTENTS

Week 1: Introduction and random variable

Objective: The student will be able to understand the importance of the course under study, and different types random variable.

Description: The course outline will be introduced with emphasis on the objectives and delivery strategies, importance and expectation for the study of the course. Different types of random variables, how to generate random variables.

Study Questions:

1. What is a random variable?
2. Distinguish between Categorical variable and Numerical variable?
3. Distinguish between Discrete variable and Continuous variable?

Reading List:

- Oyejola, B. A. and Adebayo, S. B. (2004). Basic Statistics for Biology and Agriculture Students. OLAD Ilorin.
- Lioyd R. Jaisingh (2000). Statistics for the Utterly Confuse, McGraw-Hill, USA.
- Ross, S. M. (2004). Introduction to Probability and Statistics for Engineers and Scientist. Elsevier Academic Press, USA

Week 2: Probability

Objective: The main objective is for the student to know the basic concepts of probability.

Description: Experiment, Outcomes, Trial, Chance, Event, Sample Space, and Probability Space will be defined and illustrated. Examples on tossing of coins, dice and others will be used for illustration.

Study questions:

1. What is an experiment?
2. Distinguish between Sample Space and Probability Space?
3. Distinguish between Sample Space and Universal Set?
4. Obtain the Sample space of the following:
    i. Throwing two coins together?
    ii. Throwing three coins together?
    iii. Throwing two dice together?
    iv. Throwing three dice together?

Reading List:

- Oyejola, B. A. and Adebayo, S. B. (2004). Basic Statistics for Biology and Agriculture Students. OLAD Ilorin.
- Lioyd R. Jaisingh (2000). Statistics for the Utterly Confuse, McGraw-Hill, USA.
- Ross, S. M. (2004). Introduction to Probability and Statistics for Engineers and Scientist. Elsevier Academic Press, USA

Week 3: Random Variables in Probability

Objective: The main objective is for the student to know how to generate random variables in probability.

Description: Random variable: types of random variable, probability mass function. There will be illustrated using tossing of coin or die and empirical data.

Study questions:
Obtain the probability mass function for each of the following experiments:

(i) The number of heads in throwing two coins together?
(ii) The number of heads in throwing three coins together?
(iii) The sum of the numbers that appears in throwing two dice together?
(iii) The sum of the numbers that appears in throwing three dice together?
(iv) The absolute difference of the numbers that appears in throwing two dice together?
(v) The product of the numbers that appears in throwing two dice together?

Reading List:

- Oyejola, B. A. and Adebayo, S. B. (2004). Basic Statistics for Biology and Agriculture Students. OLAD Ilorin.
- Lioyd R. Jaisingh (2000). Statistics for the Utterly Confuse, McGraw-Hill, USA.
- Ross, S. M. (2004). Introduction to Probability and Statistics for Engineers and Scientist. Elsevier Academic Press, USA

Week 4: Mathematical Expectations in Probability

Objective: The main objective is for the student to know Mathematical expectations, variances, standard deviation and their properties.

Description: Definition of Mathematical expectations, Variances, Standard deviation with their properties. Both discrete and continuous types. Relationship between probability density function and Probability distribution function.

Study questions:
If X and Y are independent random variables and ***t*** a constant, expand the following

1. E(X+2tY)

2. E(X+t)
3. E(3tXY)
4. V(tX+Y)
5. V(X+t)

Reading List:

- Oyejola, B. A. and Adebayo, S. B. (2004). Basic Statistics for Biology and Agriculture Students. OLAD Ilorin.
- Lioyd R. Jaisingh (2000). Statistics for the Utterly Confuse, McGraw-Hill, USA.
- Ross, S. M. (2004). Introduction to Probability and Statistics for Engineers and Scientist. Elsevier Academic Press, USA

Week 5: Probability distributions

Objective: The main objective is for the student to know different probability distributions that we have, how and when to use each of them.

Description: The two groups of Probability distributions: Discrete and continuous, will be highlighted. Some of the discrete probability distributions will be considered in details here, to achieve this, we consider the following:

Bernoulli and Binomial: Emphasis will be on their properties, derivation of their means and variances as well as applications of their distributions.

Study questions: a. Define a random variable that follows a Bernoulli distribution.

b. Show that the mean of a Binomial distribution is *np* and variance is *npq*?

c. If X is binomially distributed with n=6 and p=0.4, find the probability that

(i) $P(X = x)$

(ii) $P(X = 4)$

(iii) $P(x \leq 2)$

(iv) $P(2 \leq x \leq 4)$

(v) $P(x \geq 3)$

Reading list:

- Oyejola, B. A. and Adebayo, S. B. (2004). Basic Statistics for Biology and Agriculture Students. OLAD Ilorin.
- Lioyd R. Jaisingh (2000). Statistics for the Utterly Confuse, McGraw-Hill, USA.
- Ross, S. M. (2004). Introduction to Probability and Statistics for Engineers and Scientist. Elsevier Academic Press, USA

Week 6: Discrete Probability distributions

Objective: The main objective is for the student to know some discrete probability distributions that are useful in their field of study.

Description: Two other discrete probability distributions will be considered in details here, to achieve this, we consider the Poisson and Geometric with emphasis on their properties, means and variance as well as their applications in real life situations.

Study questions:

a). Define a random variable that follows a Poisson distribution with parameter(Θ).

b). What are the properties of a Poisson distribution?

c). Define a random variable that follows a Geometric distribution.

d). Suppose that particles are emitted from a radioactive source and that the number of particles (x) emitted during a one-hour period has a Poisson distribution with parameter θ. If

$$p(x=2)=\frac{2}{3}p(x=1),$$

Evaluate the (i) P(X=3)

(ii) P(X=0)

(iii) P(X≤1)

Reading list:

- Oyejola, B. A. and Adebayo, S. B. (2004). Basic Statistics for Biology and Agriculture Students. OLAD Ilorin.
- Lioyd R. Jaisingh (2000). Statistics for the Utterly Confuse, McGraw-Hill, USA.
- Ross, S. M. (2004). Introduction to Probability and Statistics for Engineers and Scientist. Elsevier Academic Press, USA

Week 7: Continuous Probability distributions

Objective: The main objective is for the student to know some useful probability distributions and when to use each of them.

Description: Normal probability distribution: its properties and applications.

Study Questions:

a). Define a random variable that follows a Normal distribution?

b). The average age of Nigerian University's Vice Chancellors are normally distributed with mean 55 and variance 81. If a Vice Chancellor is chosen at random what is the probability that his age is

(i). more than 52 years

(ii). less than 48 years

(iii). between 40 and 64 years.

(iv). In a sample of 97 Vice Chancellors, how many will be expected to be more than 63 years?

c). Find the two age limits that will be expected to contain 85% of the Vice Chancellors in the Nigerian Universities?

Reading List:

- Oyejola, B. A. and Adebayo, S. B. (2004). Basic Statistics for Biology and Agriculture Students. OLAD Ilorin.
- Lioyd R. Jaisingh (2000). Statistics for the Utterly Confuse, McGraw-Hill, USA.
- Ross, S. M. (2004). Introduction to Probability and Statistics for Engineers and Scientist. Elsevier Academic Press, USA

Week 8: Bivariate Probability distributions

Objective: The main objective is for the student to know what Bivariate probability distribution is, and when to use it.

Description: Bivariate probability distribution, marginal probability distribution.

Study Questions:

a). Define a bivariate probability distribution (f(X, Y)?

b). Define a marginal probability distribution of X?

c). Define a marginal probability distribution of Y?

d). Define a conditional probability distribution of X given Y?

Reading List:

- Oyejola, B. A. and Adebayo, S. B. (2004). Basic Statistics for Biology and Agriculture Students. OLAD Ilorin.
- Lioyd R. Jaisingh (2000). Statistics for the Utterly Confuse, McGraw-Hill, USA.
- Ross, S. M. (2004). Introduction to Probability and Statistics for Engineers and Scientist. Elsevier Academic Press, USA

Week 9: Bivariate Probability distributions
Objective: The main objective is for the student to be able to obtain the conditional probability distribution and when to use it.
Description: conditional probability, mean, variances, covariances COV(X,Y), correlation ρ(X,Y).

Study Questions:
a). Define a conditional probability distribution of Y given X?

b). Let the relationship between variables X and Y is shown in the joint probability table below:

X/ Y	1	2	3
1	0.1625	0.125	0.025
2	0.10	0.15	0.05
3	0.125	0.075	0.1875

Obtain the conditional probability f(y=3/ x=1)
c). Given the joint probability density function $f(x, y) = 4xy, \quad 0 < x < 1 \quad 0 < y < 1,$
Find the f(x/y) and f(y/x)

Reading List:
- Oyejola, B. A. and Adebayo, S. B. (2004). Basic Statistics for Biology and Agriculture Students. OLAD Ilorin.
- Lioyd R. Jaisingh (2000). Statistics for the Utterly Confuse, McGraw-Hill, USA.
- Ross, S. M. (2004). Introduction to Probability and Statistics for Engineers and Scientist. Elsevier Academic Press, USA

Week 10: Linear Regression
Objective: The main objective is for the student to know what linear regression is, how to obtain the equation and when to use it.
Description: Simple linear regression, assumptions, estimation of parameters.

Study Questions:
a). Define a simple linear regression?

b). What are the assumptions of a simple linear regression?

c). Y 60 40 72 63 36 47 55 49 44 42 68 59 64 50 63
X 120 118 128 145 116 160 167 159 139 147 150 145 140 190 188
Fit a simple linear regression model to the data and obtain the fitted values for all.

Reading List:

- Oyejola, B. A. and Adebayo, S. B. (2004). Basic Statistics for Biology and Agriculture Students. OLAD Ilorin.
- Lioyd R. Jaisingh (2000). Statistics for the Utterly Confuse, McGraw-Hill, USA.
- Ross, S. M. (2004). Introduction to Probability and Statistics for Engineers and Scientist. Elsevier Academic Press, USA

Week 11: Analysis of Variance for Simple Linear Regression

Objective: The main objective is for the student to know how to test for adequacy of regression equation and test significance of the parameters.

Description: Analysis of variance for the Simple linear regression. T – test for testing significance of parameter.

Study Questions:

a). Given the following data.

Y	60	40	72	63	36	47	55	49	44	42	68	59	64	50	63
X	120	118	128	145	116	160	167	159	139	147	150	145	140	190	188

Fit a regression equation to the data and test its adequacy?

Reading List:

- Oyejola, B. A. and Adebayo, S. B. (2004). Basic Statistics for Biology and Agriculture Students. OLAD Ilorin.
- Lioyd R. Jaisingh (2000). Statistics for the Utterly Confuse, McGraw-Hill, USA.
- Ross, S. M. (2004). Introduction to Probability and Statistics for Engineers and Scientist. Elsevier Academic Press, USA

Week 12: Correlation

Objective: The main objective is for the student to know what correlation is, how to calculate it and when to use it.

Description: Definition of correlation, assumptions and properties. Pearson's and Rank correlation.

Study Questions:

a). Define a correlation coefficient?

b). What are the assumptions of correlation coefficient?

c).

Y	60	40	72	63	36	47	55	49	44	42	68	59	64	50	63
X	120	118	128	145	116	160	167	159	139	147	150	145	140	190	188

Obtain the correlation coefficient?

Reading List:

- Oyejola, B. A. and Adebayo, S. B. (2004). Basic Statistics for Biology and Agriculture Students. OLAD Ilorin.
- Lioyd R. Jaisingh (2000). Statistics for the Utterly Confuse, McGraw-Hill, USA.
- Ross, S. M. (2004). Introduction to Probability and Statistics for Engineers and Scientist. Elsevier Academic Press, USA

Week 13: Measure of Attribute

Objective: The main objective is for the student to know some measures of Attributes, how to calculate and when to use them.

Description: Contingency table, Chi-square statistic, observed and expected frequencies. Test of association.

Study Questions:

a). Define a contingency table?

b). Given a **3 x 3** contingency table

Observed frequency

$o_{11} = 15$	$o_{12} = 20$	$o_{13} = 18$
$o_{21} = 22$	$o_{22} = 25$	$o_{23} = 21$
$o_{31} = 17$	$o_{32} = 18$	$o_{33} = 16$

Obtain **chi-square** X^2 for the table?

Reading List:

- Oyejola, B. A. and Adebayo, S. B. (2004). Basic Statistics for Biology and Agriculture Students. OLAD Ilorin.
- Lioyd R. Jaisingh (2000). Statistics for the Utterly Confuse, McGraw-Hill, USA.
- Ross, S. M. (2004). Introduction to Probability and Statistics for Engineers and Scientist. Elsevier Academic Press, USA

Week 14: Class Test

Description: The students will be assessed on the whole course for 1 hour.

Week 15: Revision/ Tutorial Exercises

Description: Solution to the test will be considered with other tutorial questions.

Study Questions:

a). Suppose that particles are emitted from a radioactive source and that the number of particles (x) emitted during a one-hour period has a Poisson distribution with parameter θ=4. Evaluate the P(X=0)?

b). If $f(x) = \frac{8}{x^3}, \quad x > 2$. Evaluate the P(X≤4)?

c). Given the $f(x) = kx(1-x\ ), 0 < x < 1$, find the value of k that will make it a probability density function?

d). Given $f(x) = \begin{cases} \frac{x^2}{91}, & x = 1,2,3,4,5,6. \\ 0, & if\ otherwise \end{cases}$

Evaluate the mean of x (E(X)).

e). If X has a Uniform distribution over the interval [3, 10]. Obtain the f(x)

f). A die is tossed 50 times. The following table gives the six numbers (**x**) and their frequency (**f**) of occurrence:-

x	1	2	3	4	5	6
f	7	9	8	7	9	10

Find the probability that a 4 appears

More Questions

QUESTION 1

a). If x is Poisson with parameter λ, and y is also Poisson with parameter θ, What is the distribution of U=x + y, if x and y are independent random variables.

b). Define a Binomial distribution?

c). Obtain the mean and the variance of a Binomial distribution?

QUESTION 2

a). Define a random variable x that follows a Continuous Uniform distribution?

b). If X follows a Poisson distribution with parameter λ=2.93, find the probability that

(i) P(X = x) (ii) P(X ≤ 1) (iii) P(x < 3)

(iv) P(1 < x < 4) (v) P(x > 2)

QUESTION 3

a). Define a random variable x that follows a Geometric Distribution?

b). What are the relationships between a Bernoulli distribution and a Binomial distribution?

c). The average age of Nigerian University's Vice Chancellors are normally distributed with mean 57 and variance 225. If a Vice Chancellor is chosen at random what is the probability that his/her age is

(i). more than 52 years

(ii). less than 48 years

(iii). between 40 and 64 years.

d). Find the two age limits that will be expected to contain 85% of the Vice Chancellors in the Nigerian Universities based on information in (c)?

QUESTION 4

a. A shop is supplied with the goods manufactured by three factories whose relative quotes are: Factory I is 30%, Factory II is 50% and Factory III is 20%. The percentages of defective goods manufactured by these factories are 3%, 2% and 5% respectively. Given that an article purchases at random at the shops will turn out to be a defective item, What is the probability that it is from
   (i) factory I. (ii) factory II (iii) factory III.

b. Obtain the mean and variance of a continuous Uniform distribution?

QUESTION 5

a. Given that

$$f(x,y)=\begin{cases}\frac{1}{7}(x+3y), 0<x<1, & 0<y<2\\ 0 \quad if \quad otherwise\end{cases}$$

Obtain the

(i) $f_1(x)$ (ii) $f_1(y)$ (iii) $f(x/y)$ (iv) $f(y/x)$

b. Define a random variable x that follows a Normal distribution?

c. Obtain the E(X) and V(X) of a Geometric distribution?